Engineering Notes

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Harmonic Representation of Aerodynamic Lift and Drag Coefficients

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Introduction

MODERN advances in highly maneuverable and thrust-vectoring aircraft designs resulted in the need for highly sophisticated mathematical models of aircraft dynamics that are accurate and applicable within the wide range of incidence angles. Such a model could not be developed without corresponding lift- and drag-force representations that will be both accurate and relatively simple. At the same time, simple and accurate models of aircraft dynamics are highly necessary for flight control and navigation system research and development. Although conventional and currently widely used linear approximations of lift and drag coefficients deliver good results for the small incidence angle, they expectedly fail long before the incidence angle even reaches its stall value [1,2].

Several harmonic approximations of lift and drag coefficients are compared and analyzed in this Note. The best representation in terms of accuracy and simplicity is found and proposed, and the problem of its parameters estimation is solved.

Aerodynamic Forces and Coefficients

Let us consider two reference systems with origin O at the center of gravity. Fixed-body system XOY has the X axis pointing forward and the Y axis pointing up. Velocity reference system $X_V O Y_V$ has the X axis always oriented along the aircraft velocity vector $\boldsymbol{V}$, as shown in Fig. 1.

Incidence α is an angle between velocity vector $\boldsymbol{V}$ and the fixed-body X axis. Lifting force R_L is directed upwards along the Y_V axis, and aerodynamic drag R_D is directed along the X_V axis. For the essentially subsonic velocities, aerodynamic lifting and drag forces are usually calculated as

$$R_L = \frac{\rho V^2 S}{2} C_L(\alpha), \qquad R_D = \frac{\rho V^2 S}{2} C_D(\alpha) \tag{1}$$

where $C_L(\alpha)$ is the lift coefficient, $C_D(\alpha)$ is the drag coefficient, ρ is the air density at the given flight altitude, and S is the characteristic wing area. Both aerodynamic lift and drag coefficients are experimentally measured during wind-tunnel tests and strongly depend on the aircraft geometry.

Using the conventional approach and assuming small incidence angles, the lifting coefficient is usually linearly approximated as [1,2]

$$C_L(\alpha) = C_L^\alpha(\alpha_0 + \alpha) \tag{2}$$

where α_0 is the zero-lift incidence angle, and C_L^α is the lift slope coefficient. Alternatively, the following straightforward linear form is sometimes used:

$$C_L(\alpha) = C_{L0} + C_{L1}\alpha \tag{3}$$

where C_{L0} and C_{L1} are some dimensionless approximation parameters. In both cases, two independent parameters are used to represent the lift coefficient.

For the same small incidence angles, the aerodynamic drag Q2
coefficient is represented in the following form:

$$C_D(\alpha) = C_{D0} + C_{D1}C_L^2(\alpha) \tag{4}$$

where C_{D0} is the lift-independent drag coefficient, and C_{D1} is the lift-induced drag coefficient.

Because all of the preceding approximations (2–4) were derived for the small incidence angles, they are not suitable for the whole incidence angle (360-deg) modeling. In this case, other approximations need to be proposed and investigated.

Different Harmonic Approximations

Considering the fact that all of the aerodynamic coefficients are *naturally* 2π *periodic*, harmonic approximations using sine and/or cosine functions are the first and obvious choice.

The lift coefficient approximations investigated in this paper are as Q3
follows:

Polynomial:

$$C_L = l_0 + \sum_{i=1}^{n} l_i \alpha^i \tag{5}$$

Sine-cosine:

$$C_L = l_0 + \sum_{i=1}^{n/2} [l_{2i-1}\sin(i\alpha) + l_{2i}\cos(i\alpha)] \tag{6}$$

Sine:

$$C_L = l_0 + \sum_{i=1}^{n} l_i \sin(i\alpha) \tag{7}$$

Even sine-cosine:

$$C_L = l_0 + \sum_{i=1}^{n/2} [l_{2i-1}\sin(2i\alpha) + l_{2i}\cos(2i\alpha)] \tag{8}$$

Even sine:

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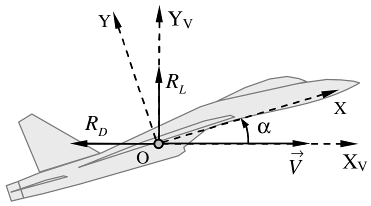

Fig. 1 Aerodynamic lift and drag forces.

$$C_L = l_0 + \sum_{i=1}^{n} l_i \sin(2i\alpha) \tag{9}$$

The drag coefficient approximations investigated in this paper are as follows:

Polynomial:

$$C_D = d_0 + \sum_{i=1}^{n} d_i \alpha^i \tag{10}$$

Sine-cosine:

$$C_D = d_0 + \sum_{i=1}^{n/2} [d_{2i-1} \sin(i\alpha) + d_{2i} \cos(i\alpha)] \tag{11}$$

Cosine:

$$C_D = d_0 + \sum_{i=1}^{n} d_i \cos(i\alpha) \tag{12}$$

Even sine-cosine:

$$C_D = d_0 + \sum_{i=1}^{n/2} [d_{2i-1} \sin(2i\alpha) + d_{2i} \cos(2i\alpha)] \tag{13}$$

Even cosine:

$$C_D = d_0 + \sum_{i=1}^{n} d_i \cos(2i\alpha) \tag{14}$$

where n is the number of harmonic coefficients used to approximate either lift or drag, α is the incidence angle in radians, and l_i and d_i are the independent parameters for the lift and drag, respectively, that are to be determined based on the best fit of experimental data. Polynomial approximation is included here for comparison purposes only.

Harmonic Approximations Evaluation

All of the functions (5–14) were compared against approximation quality criteria. The desired function is required to exhibit the following features: 1) using as few approximation parameters as possible and 2) providing accurate enough approximations for all possible incidence angles, ranging from −180 to 180 deg.

It is also appears to be reasonable that the function of choice should have better approximation quality for small angles (near zero) compared with its quality at high incidence angles. Given the experimentally measured aerodynamic coefficients, the approximation error function E is calculated using the following formula:

$$E(C, n) = \frac{1}{N} \sum_{i=1}^{N} \frac{k}{k + \alpha_i} |C(\alpha_i) - C_i^*| \tag{15}$$

where C_i^* is the experimentally measured either lift or drag coefficient at given incidence angle α_i, N is the number of experimental data points, and k is the weighting factor that is responsible for reduced error yield at high incidence angles. The actual experimental data that were used for the approximation quality evaluation had been published in [3] as the typical aerodynamics of a modern jet fighter, such as the F-15, Su-27, etc. Every approximation

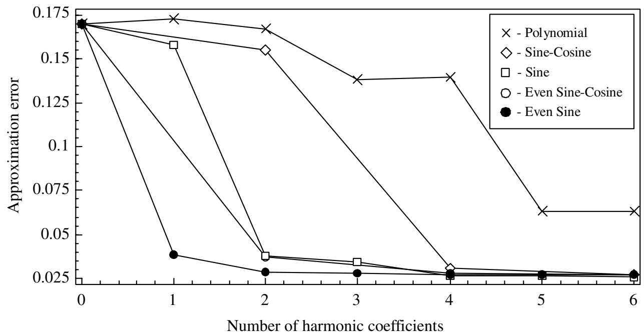

Fig. 2 Lift approximation results.

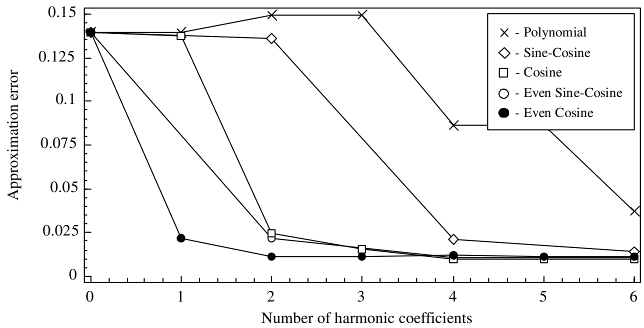

Fig. 3 Drag approximation results.

function (5–14) was best fitted into the experimental data using the least-squares method and then evaluated using Eq. (15). Calculated values of approximation error function (15) for different approximation formulas and different numbers of harmonic terms are presented in Figs. 2 and 3.

Polynomial approximations were fitted here to the best approximation within the whole angle range, rather than for small zero-incidence angles. Analyzing the obtained results, one can see that the best approximation quality is delivered by the even sine function (9) for lift and the even cosine function (14) for drag.

Approximation Parameters Calculation

From the approximation evaluation shown earlier, one can see that both of the even functions (9) and (14) deliver quite good approximation with only one or two harmonic terms, thus requiring only two or three independent parameters l_i and d_i to be identified:

$$\begin{aligned} C_L(\alpha) &= l_0 + l_1 \sin(2\alpha) + l_2 \sin(4\alpha), \\ C_D(\alpha) &= d_0 + d_1 \cos(2\alpha) + d_2 \cos(4\alpha) \end{aligned} \tag{16}$$

Another benefit of representations (16) comes from the ability to fit or identify its parameters in a way that is the most appropriate for the given problem. For example, if greater accuracy for small incidence angles is required compared with the whole-range accuracy, only the near-zero data could be used for fitting. Such a fitting is shown in Figs. 4 and 5 for lift and drag, respectively. Note that the whole-range accuracy in this case still remains acceptable.

On the other hand, the approximation parameters l_i and d_i can be identified from the commonly used linear representations (2) and (4), provided that its parameters are known. In this case, by means of Taylor series expansion of Eq. (16), the lift coefficient parameters can be determined using the following dependencies:

$$l_0 = C_L^\alpha \alpha_0, \qquad l_1 = \frac{C_L^\alpha}{2(1 + 2a)}, \qquad l_2 = \frac{C_L^\alpha a}{2(1 + 2a)} \tag{17}$$

where $a = l_2/l_1$ is the dimensionless harmonic-parameters ratio, which equals zero if a single harmonic term approximation is used

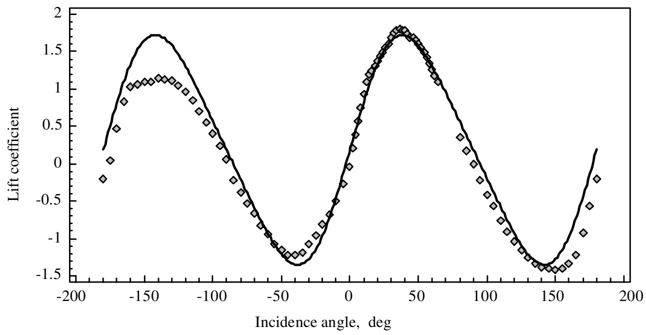

Fig. 4 Lift coefficient approximation ($l_0 = 0.1867$, $l_1 = 1.4885$, and $l_2 = 0.1991$).

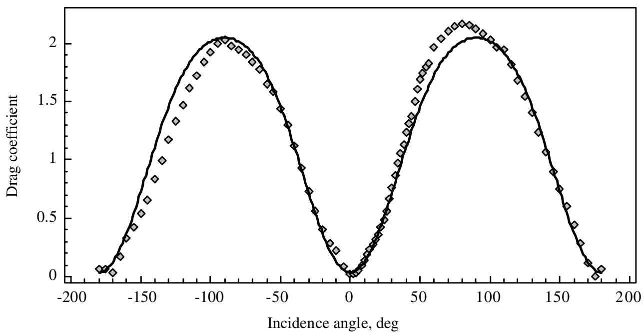

Fig. 5 Drag coefficient approximation ($d_0 = 1.1657$, $d_1 = -1.0058$, and $d_2 = -0.1253$).

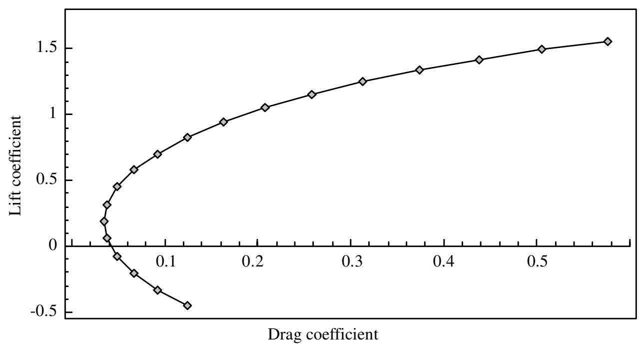

Fig. 6 Aerodynamic lift vs drag polar ($\alpha \in [-10\,\mathrm{deg},\ 26\,\mathrm{deg}]$, plotted every 2 deg).

($l_2 = 0$), or it can be simply assigned in the range from 0.1 to 0.2. Alternatively, this parameter can be identified in a more rigorous way, provided that additional aerodynamic information is available, such as the stall incidence angle, for instance.

Similarly, aerodynamic drag coefficient parameters can be calculated using the following dependencies and the known lift slope coefficient C_L^α:

$$d_0 = C_{D0} + \frac{(1+b)C_L^{\alpha^2}}{2(1+4b)} C_{D1}, \qquad d_1 = -\frac{C_L^{\alpha^2}}{2(1+4b)} C_{D1}$$

$$d_2 = -\frac{bC_L^{\alpha^2}}{2(1+4b)} C_{D1} \tag{18}$$

where $b = d_2/d_1$ is the dimensionless harmonic-parameters ratio, which is similar to the a in Eq. (17). One should note that the lift slope coefficient C_L^α can be also calculated using known harmonic lift approximation.

Despite only two harmonic terms shown in representation (16), more terms could be added for greater accuracy, as shown in Eqs. (9) and (14).

Aerodynamic Fineness and Polar

Other useful aerodynamic quantities such as aerodynamic fineness and polar could be either calculated or approximated using the harmonic representations (16), (9), or (14). The aerodynamic fineness can be calculated as

$$K(\alpha) = \frac{C_L(\alpha)}{C_D(\alpha)} = \frac{l_0 + l_1 \sin(2\alpha) + l_2 \sin(4\alpha)}{d_0 + d_1 \cos(2\alpha) + d_2 \cos(4\alpha)} \tag{19}$$

Corresponding aerodynamic lift versus drag polar is shown in Fig. 6.

Known polar data also allow determining parameters of the harmonic approximation of lift and drag coefficients representation.

Conclusions

Empirical aerodynamic lift and drag coefficients representations (16) allow expanding of existing linear representations (2) and (4) to the whole range of possible incidence angles. Such improved harmonic representations could easily replace the conventional linear representations and deliver better results, especially when extremely high incidence angles are required. As presented in the Note, harmonic approximations of aerodynamic lift and drag coefficients have been already successfully applied to highly maneuverable, next-generation, thrust-vectoring, jet fighters' flight control system analysis, simulation, and design.

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