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INVESTIGATION OF MICROMECHANICAL INERTIAL DEVICES

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Abstract
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Investigation and comparative analysis of different designing structures of the micromechanical inertial devices, recommendations development for improvement including performances optimisation are seem to be actual on the threshold of mass producing of micromechanical inertial devices. Mathematical models development results for the different designing structures of micromechanical angular rate sensors are presented in this report. The list of requirements and properties for their structure elements is determined on the basis of obtained results.

Introduction

The gyros and gyroscopic devices are main primary information sensors of angular and non-angular displacements of the moveable objects in systems of navigation, stabilisation and guidance. Using of microelectronics technologies for creating of miniature gyros became possible with their development. Micromechanical devices producing technology is practically perfect in the companies leading at this branch at present. Therefore, investigation and comparative analysis of different designing structures of the micromechanical inertial devices, recommendations development for improvement including performances optimisation are seem to be actual on the threshold of mass producing of micromechanical inertial devices.

There are no doubts in availability of the micromechanical devices today. Among factors that have influence on advantages of such sensors to sensors of other types we can detach tiny size, low costs during mass production, low power cost. All of those factors objective promote to widespread using of the micromechanical inertial devices. Custom-made performances of the micromechanical sensors cause their using in non-traditional for inertial devices kinds of products. From other hand accessibility of microelectronics technologies cause possibility of accelerated mastering in manufacturing and mass production.

There are many structures for development and designing of the micromechanical angular rate sensors at present. Let's consider in detail two of them: gimballed and tuning-fork.

Gimballed micromechanical gyro

One of the most well known embodiment example of gimballed structure is gimballed vibrating gyro (GVG) developed in The Charles Stark Draper Laboratory, Inc. [1]. Gimballed micromechanical gyro doesn't have any rotational elements and instead them vibration of its elements around torsion's axes is used. The gyro can be represented by double gimbals structure (Fig.1) with massive element (proof mass) mounted on the inner frame of the gimbals. The inner frame can be represented as gyroscopic element and the outer frame as drive. Both frames are connected by means of torsion elements. The torsions have small stiffness on rotation in comparison with stiffness on flexure. The outer frame oscillates from excitation system with small amplitude and high frequency around torsion's axes. In the same time the inner frame became sensible to angular rate along axis that is orthogonal to the gyro's plane. It oscillates around its torsion's axis with amplitude depended from the outer angular rate.

Excitation of the outer frame's oscillation may be performed by means of electrostatic forces, and recording of output signal - with a help of capacitors. Electrodes are vacuum deposited on surfaces of the outer and inner frames.

From the mathematical modelling point of view micromechanical gimballed gyro has two degrees of freedom. As generalised coordinates let's choose the rotation angle of the outer frame relatively of basis and rotation angle of the inner frame relatively outer frame. Let's designate such generalised coordinates as β and α respectively.

For the arbitrary oriented quasi-constant angular rate vector Ω of basis rotation motion's equations of the micromechanical gimballed gyro will be of the following kind:

$$\begin{cases} I_2\ddot{\beta}+f_2\dot{\beta}+c_2\beta+G\dot{\alpha}\left(\Omega_z+\beta\Omega_x\right)+ \\ +D_0\left[2\dot{\alpha}\left(\dot{\beta}+\Omega_y\right)-\Omega_x\Omega_y\right]+D_1\left[\left(\Omega_x^2-\Omega_z^2\right)\beta+\Omega_x\Omega_z\right]=M_2(t), \\ I_1\ddot{\alpha}+f_1\dot{\alpha}+c_1\alpha-G\dot{\beta}\left(\Omega_z+\beta\Omega_x\right)+ \\ +D_0\left[\alpha\left(\dot{\beta}+\Omega_y\right)^2-\alpha\Omega_z^2-\Omega_y\left(\Omega_z+\beta\Omega_x\right)\right]=0, \end{cases}$$

where $\vec{\Omega}=\left\{\Omega_x,\Omega_y,\Omega_z\right\}$ - angular rate vector need to be measured, I_1, I_2, G, D_0 and D_1 - factors that are depended from geometrical parameters of the structure. $M_2(t)$ - torque from outer forces that cause excited oscillations of the outer frame.

Demonstrated motion equations of the micromechanical gimballed gyro are non-linear and hence we need to use special approximate methods of analytical and numerical solving.

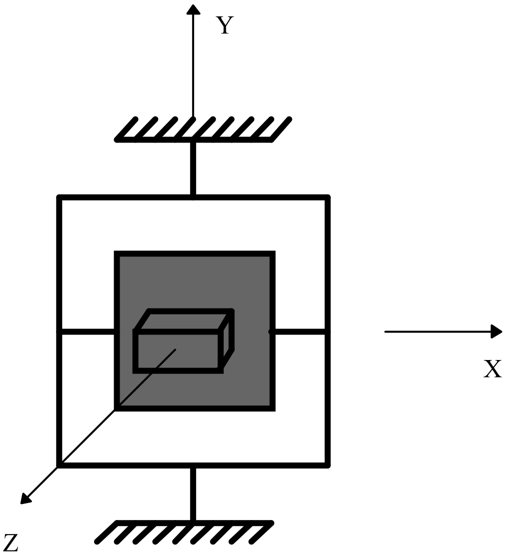

Fig.1. Principal structure of the micromechanical gimballed gyro

The effect we are interested in that give us possibilities to perform measuring of outer angular rate will be described by solution found from linearised system of equations. System's solution is considered as general while contribution of the rejected non-linear parts must be considered as harmful influence.

By assuming orthogonal to the device's plane orientation of the outer angular rate vector we can essentially reduce represented above equations:

$$\begin{cases} I_2\ddot{\beta}+f_2\dot{\beta}+c_2\beta+G\Omega\dot{\alpha}-D_1\Omega^2\beta-2D_0\dot{\alpha}\dot{\beta}\alpha=M_2(t), \\ I_1\ddot{\alpha}+f_1\dot{\alpha}+c_1\alpha-G\Omega\dot{\beta}+D_0\alpha\left(\dot{\beta}^2-\Omega^2\right)=0. \end{cases}$$

With respect to the angular rate vector's orientation motion equation are still non-linear, but all of the harmful parts are included with constant factors D_0 and D_1, that depends only from inertia performances of the structure elements. Request on reducing values of that constant factors results in possibility of optimal choosing of structure elements parameters that results in influence reducing of non-linear parts.

From other hand we have to maximise constant factor G of gyro cross-connection and this one also depends only from inertia parameters. Thus all of the mentioned above optimisation conditions can be represented as follows:

$$G=I_{x2}+I_{y2}-I_{z2}\rightarrow \max,$$

$$D_0=I_{y2}-I_{z2}\rightarrow \min,$$

$$D_1 = I_{x1} + I_{x2} - I_{z1} - I_{z2} \to \min.$$

One of the optimisation ways consist in choosing of the linear sizes of the proof mass mounted on the inner frame of the gimballed micromechanical gyro. Performed calculations for the different materials of the proof mass with respect to given sizes for all other elements of gyro have resulted in necessity of producing the mass in shape of parallelepiped with length to width ratio thereabout 5..6 (depends from proof mass material).

With choice of the geometrical parameters of the device with respect of optimisation conditions its behaviour will be rather precisely described by linear system of the differential equations that looking like [2,3]:

$$\begin{cases} I_2\ddot{\beta} + f_2\dot{\beta} + c_2\beta + G\Omega\dot{\alpha} = M_2(t), \\ I_1\ddot{\alpha} + f_1\dot{\alpha} + c_1\alpha - G\Omega\dot{\beta} = 0, \end{cases}$$

or after division of the first equation on I_2 and the second one on I_1

$$\begin{cases} \ddot{\beta} + 2h_2\dot{\beta} + \omega_{02}^2\beta + g_2\Omega\dot{\alpha} = m_2(t), \\ \ddot{\alpha} + 2h_1\dot{\alpha} + \omega_{01}^2\alpha - g_1\Omega\dot{\beta} = 0. \end{cases}$$

In absence of the angular rate oscillations on α angle are absent too. Otherwise the amplitude of the inner frame oscillations on frequency of excitation will be determined by expression

$$A_\alpha(\omega) = \frac{g_1\omega\Omega}{\sqrt{\Delta(\omega,\Omega)}},$$

$$\Delta(\omega,\Omega) = \left(\omega^2 - \omega_{01}^2\right)^2\left(\omega^2 - \omega_{02}^2\right)^2 + 2\omega^6\left[2h_1^2 + 2h_2^2 - g_1g_2\Omega^2\right] +$$
$$+\omega^4\left[\left(4h_1h_2 + g_1g_2\Omega^2\right)^2 + 2g_1g_2\Omega^2\left(\omega_{01}^2 + \omega_{02}^2\right) - 8h_1\omega_{02}^2 - 8h_2\omega_{01}^2\right] +$$
$$+2\omega^2\left[2h_2^2\omega_{01}^4 + 2h_1^2\omega_{02}^4 - g_1g_2\Omega^2\omega_{01}^2\omega_{02}^2\right].$$

For increasing of gyro sensitivity to the angular rate influence it is necessary to raise oscillations of the outer frame on one of the eigen frequencies of the system. It can be either eigen frequency of the outer frame's oscillations or eigen frequency of the inner frame's oscillations. The essential peculiarity of the considered circuit is dependence of the eigen frequencies of the system from the angular rate. It results in shift of the eigen frequencies in the following dependence from the angular rate:

$$\omega_{10}^{*2} = \omega_{10}^2 + g_1g_2\Omega^2 + \frac{g_1^2g_2^2}{4d_2}\Omega^4,$$

$$\omega_{20}^{*2} = \omega_{20}^2 - \frac{g_1^2g_2^2}{4d_2}\Omega^4.$$

As it is apparent from results above the greater eigen frequency ω_{10}^* shifts in positive direction and lower ω_{20}^* reduces. For the small angular rates displacement of the greater eigen frequency will be more essential than for smaller. The amplitude-frequency response (AFR) and displacement direction of the frequencies are submitted on fig. 2.

By choosing the larger eigen frequency for excitation of the system we raise the factor of amplification on measuring influence, but we reduce a linear range of measurement of angular rate and on the contrary. By exciting the system on the smaller eigen frequency, we can increase a range of linearity of the amplification factor approximately in 3 times. If the excitation system is realised by a principle of automatic observation of the resonant frequency, it is necessary to use the larger eigen frequency. The diagram on fig.3 demonstrates dependence of the amplitude of the inner frame's oscillations from the outer angular rate.

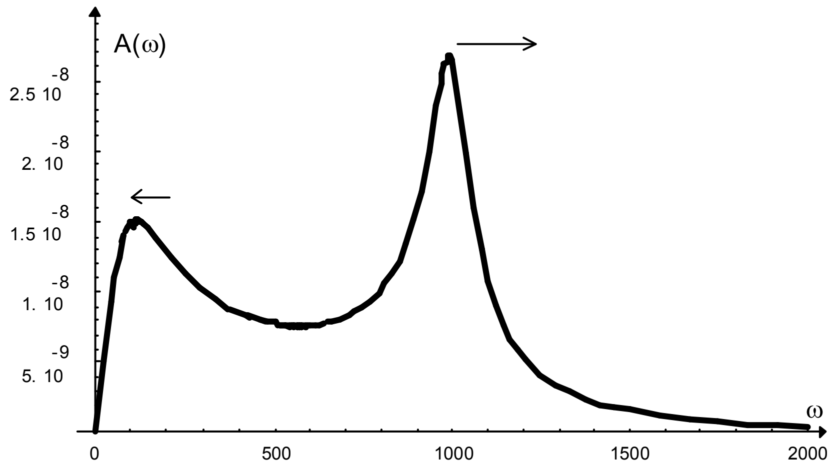

Fig.2. AFR and displacement direction of the eigen frequencies

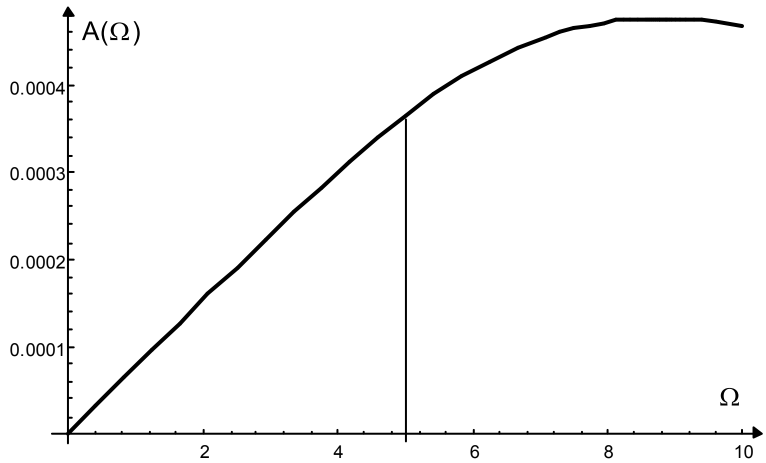

Fig.3. Amplitude from angular speed dependence

Tuning-fork micromechanical gyro

The majority from the developed at the moment micromechanical angular rate sensors use as classical tuning-fork circuit, so every possible its modifications. An example of such modification can be micromechanical tuning-fork gyro (TFG), developed in The Charles Stark Draper Laboratory, Inc. [4]. As well as considered in the previous section gimballed gyro, tuning-fork micromechanical gyro is produced from silicon by using technologies of the semi-conductor microelectronics.

In the most general case the design of the tuning-fork gyro can be represented by two inertial masses (proof masses), mounted in the elastic suspension with ensuring for each of them at least two degrees of freedom. By excitation of oscillations on one of the coordinates at the presence of the outer angular rate the oscillations in a direction of other coordinate are appear.

Let's consider a flat design, consisting from the two symmetric proof masses fixed to the basis through elastic suspension. In addition let's introduce mobile reference system XYZ so that the whole design is located in the plane YZ and the axis X - is perpendicular to it (Fig. 4).

Each of the masses can move in a direction of axes X and Y independently from each other and also together they can be turned around of axis Z on the φ angle. We shall take into account that the angular rate vector is oriented along Z axis. In this case with exciting oscillations of the masses in a direction of the Y axis at presence of the outer angular rate there will be oscillations in the X direction and angular oscillations around Z axis.

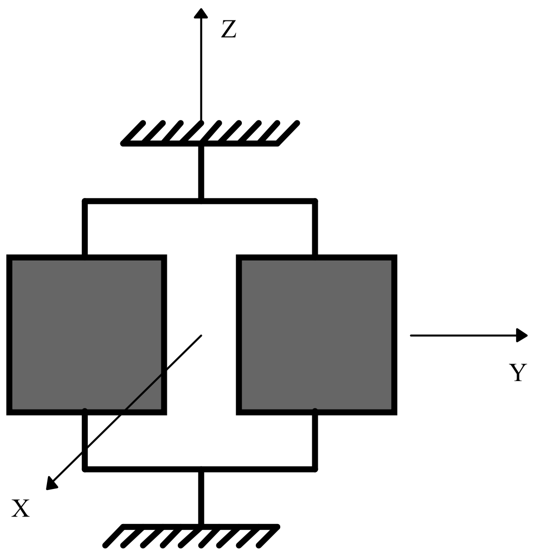

Fig.4. Principal structure of the tuning-fork gyro

The linearized system of the differential equations of the micromechanical tuning-fork gyro's movement looks like the follows

$$\begin{cases} \ddot{x}_1 + 2h_x \dot{x}_1 + \left(k_{11}^2 - \Omega_z^2\right)x_1 - k_{12}^2 x_2 - 2\Omega_z \dot{y}_1 + r_0 \ddot{\varphi} = 0, \\ \ddot{x}_2 + 2h_x \dot{x}_2 + \left(k_{22}^2 - \Omega_z^2\right)x_2 - k_{21}^2 x_1 - 2\Omega_z \dot{y}_2 - r_0 \ddot{\varphi} = 0, \\ \ddot{y}_1 + 2h_y \dot{y}_1 + \left(k_{33}^2 - \Omega_z^2\right)y_1 - k_{34}^2 y_2 + 2\Omega_z \dot{x}_1 = f_0(t), \\ \ddot{y}_2 + 2h_y \dot{y}_2 + \left(k_{44}^2 - \Omega_z^2\right)y_2 - k_{43}^2 y_1 + 2\Omega_z \dot{x}_2 = -f_0(t), \\ \ddot{\varphi} + 2h_\varphi \dot{\varphi} + \left(k_{55}^2 - f_1\right)\varphi + b_0\Omega_z\left(\dot{y}_2 - \dot{y}_1\right) - \frac{b_0}{2}\left(\ddot{x}_2 - \ddot{x}_1\right) + f_2\left(x_2 - x_1\right) = 0, \end{cases} \qquad (1)$$

where r_0 - distance from the origin of the reference system up to centre of mass of the each proof mass; b_0 - parameter of inertia; k_{ij}^2 - parameters that are proportional to stiffness of elastic connections; Ω_z - outer angular rate; h_x, h_y, h_φ - damping factors.

The amplitude of the forced oscillations of the proof masses along Y axis as function of frequency ω of the harmonic excitation $f_0(t)$ is determined by formula

$$B_1(\omega) = \frac{F_0}{\sqrt{\left(k_{34}^2 + k_{33}^2 - \Omega_z^2 - \omega^2\right)^2 + 4h_y^2\omega^2}} = -B_2(\omega),$$

where F_0 - excitation force amplitude, ω - frequency of excitation.

There are two eigen frequencies in the structure in a direction of the Y coordinate at presence of the elastic connection between proof masses: $\omega_1 = \sqrt{k_{33}^2 - k_{34}^2}$ and $\omega_2 = \sqrt{k_{33}^2 + k_{34}^2}$. The oscillations with the first frequency are correspond in-phased of the proof masses, and on the second are correspond to the antiparallel oscillations. Excitation on the greater frequency makes more simple to perform the differential design of the device and also to increase working frequency that in its turn increase transfer factor of outer angular rate.

If design of the elastic suspension don't allow turn around Z axis then amplitude of the oscillations on X coordinate from acting of the Coriolis forces is proportional to outer angular rate and can be calculated under the formula

$$A_1(\omega) = \frac{2B_1\omega}{\sqrt{\left(k_{11}^2 + k_{12}^2 - \Omega_z^2 - \omega^2\right) + 4h_x^2\omega^2}}\Omega_z = -A_2(\omega).$$

In this case by using the differential design of the device the output signal of the system proportional to the angular rate will be doubled.

Besides it, using of the differential design allows to exclude influence of linear vibration along the X axis on the frequency of excitation. As shows the analysis of the movement differential equations angular vibration with frequency of excitation on the measuring axis results in constant component in the output signal on the frequency of excitation and harmonic component on the double frequency of excitation. These components in the output signal can be excluded from it by known methods. Influence on the system of linear and angular vibrations will result in additional components in the output signal with frequencies of these vibrations. Such harmful components can easily be excluded with a help of filtration of the signal on working frequency.

If suspension besides moving of the proof masses in the X direction allows turning both masses around Z axis then the reaction of the system on the angular rate will be a little bit another: the output signal recorded from the pickoffs will be proportional not only to progressive displacement of the masses but also to their displacement from turn. There will be two components proportional to the angular rate in the output signal that will be result in increasing the transfer factor on the angular rate.

Besides the amplitudes of the progressive displacement of each proof mass will change. In this case they can be calculated through the following formulae:

$$A_1(\omega) = \mathrm{Re}\left\{\frac{2i\omega\Omega_z\overline{B}_1}{\overline{d}_x}\left[1 + \frac{\overline{d}_x\left(\omega^2 r_0 b_0\right) - \omega^4 r_0 b_0}{\overline{d}_x\overline{d}_\varphi - \omega^4 r_0 b_0}\right]\right\} \approx \frac{4\omega B_1}{\sqrt{\left(k_{11}^2 + k_{12}^2 - \Omega_z^2 - \omega^2\right)^2 + 4h_x^2\omega^2}}\Omega_z,$$

where

$$\overline{d}_x = k_{11}^2 + k_{12}^2 - \Omega_z^2 - \omega^2 + 2h_x i\omega,$$
$$\overline{d}_\varphi = k_{55}^2 - \omega^2 + 2h_\varphi i\omega.$$

As we can see in this case the amplitude of linear oscillations of the masses in the X direction is doubled in comparison with similar amplitude at absence of an opportunity of turn of the masses around Z axis.

The amplitude of angular oscillations D for brevity we shall write down with using of complex variables

$$D(\omega) = \mathrm{Re}\left\{\frac{2i\omega\overline{B}_1 b_0\left(\overline{d}_x - \omega^2\right)}{\overline{d}_x\overline{d}_\varphi - \omega^4 r_0 b_0}\right\}.$$

The resulting output signal for the each mass will be proportional to the total amplitude of linear and angular oscillations:

$$L_1(\omega) \approx A_1(\omega) - r_0 D(\omega),$$
$$L_2(\omega) \approx A_2(\omega) + r_0 D(\omega).$$

On the other hand with displacement along the X axis and turn around Z axis of the proof masses under influence of the Coriolis forces, the forces generated by excitation system will create additional parametrical disturbance expressed in the differential equations of the system (1) through harmonic factors

$$f_1(t) = b_0 f_0(t) \text{ and } f_2(t) = \frac{b_0}{2r_o} f_0(t).$$

Presence of such additional torques besides the harmful contribution to the output signal can result in parametrical excitation that will result in loss of serviceability of the device. It is possible to specify two ways of reduction of probability of such outcome. First consists in using of the force rebalance circuit of the device that principally will exclude appearance of the additional torques from the excitation system. The second way requires a choice of parameters of the elastic elements of the suspension so that they have to satisfy to the following condition: any of the eigen frequencies of the system that appropriate to displacement of the proof masses on x and φ coordinates should not be equal to the half of excitation frequency.

For increasing of the angular rate transfer factor the eigen frequencies of the system to X axis displacements of the proof masses have to be chosen near to the excitation frequency. By neglecting of damping these frequencies can be approximately obtained from the following polynomial equation:

$$\omega^4\left(1 + 2b_0 r_0\right) - \omega^2\left(k_{11}^2 + k_{12}^2 + k_{55}^2 - \Omega_z^2\right) + k_{55}^2\left(k_{11}^2 + k_{12}^2 - \Omega_z^2\right) = 0.$$

As we can see the eigen frequencies of the system that appropriate to the output coordinates depend from the angular rate. In result we shall observe displacement of the eigen frequencies in comparison with their values on the motionless basis. It will result in the variable angular rate amplification factor that reduces performance of the sensor. One of possible ways to eliminate this lack is creation of the excitation system with automatic tuning of the working frequency. In this case it is possible to perform measurements with constant amplitude of the excited oscillations.

Conclusions

On the basis of performed researches of the various design structures of the micromechanical inertial devices, their mathematical models, we can formulate the following recommendations for the devices development using the gimballed and tuning-fork types of design circuits:

- One of the possible optimisation of the micromechanical gimballed gyro consists in a choice of the sizes of the proof mass so that its length has to be nearly in 5-6 times (depending on a material of mass) greater than width.
- Choosing the greater eigen frequency of the gimballed circuit as excitation frequency results in increasing of the angular rate amplification factor but decreasing of the linear range for this factor approximately in 3 times.
- It is more simple to perform antiparallel oscillation's excitation of the masses in the device based on the tuning-fork circuit on the greater eigen frequency and this allows more simple to perform the differential circuit of the device, and also increasing of the working frequency results in increasing of the angular rate transfer factor.
- Using the differential circuit allows to exclude influence of the linear vibration on the excitation frequency along the output axis.
- Influence on the system of linear and angular vibrations will result in additional components in the output signal with frequencies of these vibrations. Such harmful components can easily be excluded with a help of filtration of the signal on the working frequency.
- For eliminating of the parametrical resonance in the tuning-fork circuit with possibility of rotation of the masses around measuring axis is expediently to use the force rebalance design circuit and don't allow concurrence of the double eigen frequencies of the system with frequency of excitation.

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