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Optimal Filtering of Temperature Errors for Coriolis Vibratory Gyroscopes

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***Abstract*— optimal static filter of stochastic temperature disturbances is proposed in this paper. Filter is based on the model of the temperature variations influences via undesired cross-damping. Instead of temperature measurements its power spectral density is used along with the corresponding model of the moving object dynamics.**

Keywords—Coriolis, vibratory, gyroscope, MEMS, temperature errors

I. Introduction

One of the emerging application for Coriolis vibratory gyroscopes (CVG) is the development of miniature navigation and control systems for unmanned aerial vehicles (UAV). Providing significant miniaturization opportunities when fabricated using micro-machining technologies, CVG are commonly referred to as micro-electro-mechanical system (MEMS) gyroscope [1, 2]. As such, CVGs are used as an angular rate sensor along with accelerometers and magnetometers to provide attitude measurements in many different applications, including small UAVs [3]. Unfortunately, comparing to other implementations of CVGs, MEMS gyroscopes have relatively low scale factor and bias stability under influence of the operational environment [4]. One of the factors capable of producing zero rate output is temperature variations via cross-damping. Angular rate errors due to the temperature variations can be efficiently reduced if accurate temperature measurements are available on-board [5]. However, suitable temperature measurements are often not available in many systems, especially in miniature ones. This paper presents the research on possibility to improve performances of MEMS CVGs using statistical characteristics of the temperature influences in case when direct temperatures are not available.

II. Dynamics of the Sensitive Element

Motion equations of the CVG sensitive element in terms of amplitude-phase complex variables

$$A_i(t) = A_{i0}(t)e^{j\varphi_{i0}(t)},$$

where i equals 1 or 2 for the primary or secondary oscillations correspondingly, A_{i0} and φ_{i0} are the amplitudes and phases of the primary and secondary oscillations, and with respect to the cross damping and slowly varying amplitudes of secondary oscillations ($\ddot{A}_2 \approx 0$) can be written as

$$\begin{aligned} &2(\zeta_2 k_2 + j\omega)\dot{A}_2 + (k_2^2 - \omega^2 + 2j\omega k_2\zeta_2)A_2 \\ &= (j\omega g_2\Omega + 2j\omega\zeta_{12}k_1 + \dot{\Omega})A_1. \end{aligned} \quad (1)$$

Here k_1 and k_2 are the corresponding natural frequencies, ζ_1 and ζ_2 are the dimensionless relative damping coefficients, ζ_{12} is the cross-damping coefficient between primary and secondary oscillations, Ω is the measured angular rate, which is orthogonal to the axes of primary and secondary motions, g_2 is the CVG design specific coefficient, ω is the excitation frequency of the primary oscillations, and

$$A_1 = \frac{q_{10}}{k_1^2 - \omega^2 + 2jk_1\zeta_1\omega}$$

is the constant (does not depend on time) complex amplitude of the primary oscillations. Looking at (1) one should note that temperature dependent cross-damping term is indistinguishable from the Coriolis term. Applying Laplace transformation to (1) and solving for the secondary complex amplitude, one can obtain expression for the measured angular rate as

$$\Omega^*(s) = W_\Omega(s)\cdot\Omega(s) + W_\Omega^\zeta(s)\cdot\zeta_{12}(s). \quad (2)$$

Here main dynamics of CVGs is described by the simplified system transfer function

$$W_\Omega(s) = \frac{k\zeta}{s + k\zeta}, \quad (3)$$

and the erroneous component of the angular rate caused by the temperature dependent cross-damping is

$$W_\Omega^\zeta(s) = \frac{2k_2(k_2^2 - \omega^2 + 2jk_2\omega\zeta_2)}{g_2(k_2^2 - \omega^2 + 2k_2\zeta_2 s + 2j\omega(s + k_2\zeta_2))}. \quad (4)$$

Transfer function in (4) can be further simplified using the same following assumptions that were used to obtain (3) [6]: natural frequencies are equal ($k_1 = k_2 = k$) as well as relative damping coefficients ($\zeta_1 = \zeta_2 = \zeta$), and primary oscillations excitation frequency is $\omega = k\sqrt{1-2\zeta^2}$. With these assumptions transfer function (4) becomes

$$W_{\Omega}^{\zeta}(s) = \frac{2k^2\zeta}{g_2(s+k\zeta)}. \quad (5)$$

Transfer function (5) allows efficient analysis of errors due to the temperature variations via cross-damping.

III. Model of the Temperature Relatied Damping

Assuming that the cross-damping coefficient is a function of the temperature shift T from the calibration temperature, it can be approximated using polynomial as

$$\zeta_{12} = \zeta_{12}(T) \approx \sum_{i=0}^{n} \zeta_i^T T^i. \quad (6)$$

Temperature related coefficients ζ_i^T can be determined experimentally when ambient temperature is known and angular rate is absent. Nevertheless, in most of the cases we observe angular rate as the gyro output. In order to relate angular rate to the input cross damping, let us use steady state of the transfer function (5) as

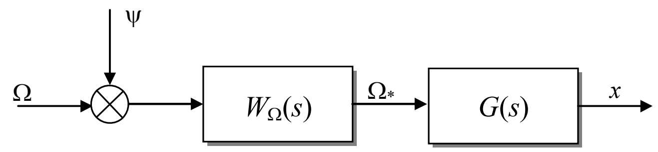
$$\Omega(T) = W_{\Omega}^{\zeta}(s \to 0)\zeta_{12}(T) \approx \frac{2k}{g_2}\sum_{i=0}^{n} \zeta_i^T T^i = \sum_{i=0}^{n} \Omega_i^T T^i. \quad (7)$$

Here the cross-damping model coefficients Ω_i^T can be determined from the experimental data.

IV. Stochasic Temperature Disturbances

Analysing (2) one should note that temperature influences output of a CVG exactly like an angular rate and the temperature related output is undistinguishable from the angular rate measurements. In this sense temperature influences should be treated as a process noise or disturbances to the CVG system (see Figure 1).

Fig. 1. CVG with added cross-damping disturbances



Here $W_{\Omega}(s)$ is the system transfer function given by (3), Ω is the input angular rate, $G(s)$ is the optimal filter yet to be developed, x is the filtered output of the system, which in ideal case is equal to the angular rate Ω, and ψ are the temperature related disturbances given by the following relation

$$\psi(s) = \frac{2k}{g_2}\zeta_{12}(s). \quad (8)$$

Looking at the system in the figure 1, one can see that the only way to separate output resulting from the angular rate, from the output generated by the temperature disturbances is to take into account additional information about sources of the angular rate and the temperature disturbances.

Assuming that CVG is installed on a maneuverable object, such as UAV or land vehicle, its power spectral density can be represented as the following low-pass model

$$S_{\Omega}(s) = \frac{\sigma^2 B^2}{B^2 - s^2}. \quad (9)$$

Here B is the object bandwidth and σ is the standard deviation of the object angular rate.

It is apparent that the temperature variations are slow and therefore could be adequately represented by the following random walk model

$$S_{\psi}(s) = \frac{\gamma^2\sigma^2}{-s^2}. \quad (10)$$

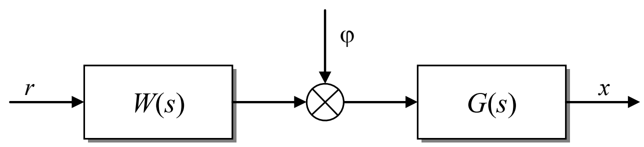
Here γ is the disturbance to angular rate power ratio ("noise-to-signal" ratio).

Power spectral densities (9) and (10) can now be used to synthesise optimal filter, reducing errors caused by the temperature variations.

V. Optimal Filter Synthesis Algorithm

Synthesis algorithm for optimal filter of stochastic disturbances for CVG as a system shown in Fig. 2 below has been presented in [7]. In the most general case, *W(s)* is the matrix of sensor transfer functions, *G(s)* is the matrix of filter transfer functions, φ is the noise vector, r is the input vector, which then is measured by the sensor, and x is the system output vector, which in our case is an estimation of the input.

Fig. 2. Optimal noise filtering



Error of this system is defined as a difference between the actual output of the system x and the ideal output, which is the given desired transformation $H(s)$ of the input:

$$\varepsilon = x - H(s) \cdot r \, .$$

It is also assumed that signals x and r are the centred stochastic processes with known spectral densities $S_{rr}(s)$, $S_{\varphi\varphi}(s)$, $S_{r\varphi}(s)$, and $S_{\varphi r}(s)$.

Performance criterion for the system is assumed in the following form:

$$J = E\{\varepsilon' \cdot R \cdot \varepsilon\} = \frac{1}{j} \int_{-j\infty}^{j\infty} tr(S'_{\varepsilon\varepsilon} \cdot R) ds \, . \qquad (11)$$

Here R is the weight matrix, and $S'_{\varepsilon\varepsilon}(s)$ is the transposed matrix of the error spectral densities. Using Wiener-Khinchin theorem we can calculate the error spectral density from the system transfer functions and signal spectral densities as follows:

$$\begin{aligned} S'_{\varepsilon\varepsilon}(s) = (GW - H)S'_{rr}(W_* G_* - H_*) + (GW - H)S'_{\varphi r} G_* \\ + GS'_{r\varphi}(W_* G_* - H_*) + GS'_{\varphi\varphi} G_*, \end{aligned} \qquad (12)$$

where "*" designates Hermite conjugate. By means of introducing new variables defined as

$$DD_* = WS'_{rr}W_* + WS'_{\varphi r} + S'_{r\varphi}W_* + S'_{\varphi\varphi} \, ,$$
$$\Gamma\Gamma_* = R \, , \quad G_0 = \Gamma G D \, ,$$
$$T = \Gamma H (S'_{rr} W_* + S'_{\varphi r}) D_*^{-1} \, ,$$

and substituting power spectral density (12) into (11), first variation of the performance criterion (11) with respect to the unknown filter related function G_0 will be

$$\delta J = \frac{1}{j} \int_{-j\infty}^{j\infty} tr[(G_0 - T)\delta G_0 + \delta G_{0*}(G_{0*} - T_*)] ds \, . \qquad (13)$$

Minimum of the performance criterion is achieved when first variation (13) is zero, which is achieved when

$$G = \Gamma^{-1}(T_0 + T_+)D^{-1} \, . \qquad (14)$$

Here T_0 is the integral part of the matrix T, and T_+ is the part of the matrix T that contains only poles with negative imaginary part. These matrices are the result of the Wiener separation procedure.

For the case of stochastic temperature disturbances, power spectral density $S_{rr}(s)$ corresponds to (10), and spectral density $S_{\varphi\varphi}(s)$ can be calculated from (9) using Wiener-Khinchin theorem as follows:

$$S_{\varphi\varphi}(s) = |W_\Omega(s)|^2 S_\psi(s) = \frac{\gamma^2 \sigma^2 k^2 \zeta^2}{-s^2(-s^2 + k^2\zeta^2)} \, . \qquad (15)$$

Spectral density (15) along with the spectral density (10) can now be used to derive optimal filters based on the formula (14). After performing all necessary transformations, the optimal filter is found in the following form:

$$G(s) = \frac{B\gamma(k\zeta + s)}{k\zeta(B\gamma + s\sqrt{B^2 + \gamma^2})} \, . \qquad (16)$$

Optimal temperature errors filter (16) can now be used to reduce effect of the temperature variations on CVG performances. It is also important to note that filter (16) is a static transfer function and therefore does not require computational devices, and can be implemented using analog electronics as an application specific integrated circuit.

VI. Conclusions

Presented above approach to synthesis of the stochastic temperature disturbances filters resulted in a static filter capable of improving the performances of Coriois vibratory gyroscopes in case of absence of temperature measurements. Statistical characteristic of the temperature disturbances, such as its power spectral density, is used instead. The further analysis of the filter parameters optimisation and studying its performances is viewed as a possible future development of the current research.

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