

УДК 681.5.015.24 (045)

## EVOLUTIONARY CONTROL ALGORITHM PARAMETERS OPTIMISATION

V. Apostolyuk

Institute of Electronics and Control Systems  
National Aviation University  
Kiev, Ukraine  
[apostolyuk@astrise.com](mailto:apostolyuk@astrise.com)

### Introduction

Simulated natural evolution being applied to the multi-parameter and multi-objective global optimisation problems have already resulted in a well established group of so-called evolutionary algorithms [1, 2]. At the same time, constantly increasing computational power of the microprocessors significantly widens areas of application for such algorithms. Application of conventional genetic algorithms to predictive control with fixed prediction horizon (PH) has been in active development during the last decade [3, 4]. Variable length genetic algorithms (VLGA) that allow implementing variable PH optimal control are still under investigation, which already uncovered certain problems. For instance, converging to an optimal solution may take significant amount of generations in comparison with the state space traversal approach [5]. One way to improve the performances of VLGA is to optimize parameters of the algorithm, such as population size and probabilities of genetic operations. Some results on search of the best such parameters are presented in this paper.

### Benchmarking control problem

Formulation of general optimal control problem is well known and here we shall give emphasis only to the aspects that are essential to the subject. Let the state of the system at time  $t$  be a vector  $\vec{x}(t) = \{x_1(t), \dots, x_n(t)\}$  in an  $n$ -dimensional Euclidean space which we shall call the state space  $X$ . The steering device or control we model as an  $m$ -dimensional vector function of time  $\vec{u}(t) = \{u_1(t), \dots, u_m(t)\}$ . The components of  $\vec{u}(t)$  are allowed to be piecewise continuous and the values they can take are bounded so that at any time  $t$ ,  $\vec{u}(t)$  lies in some bounded region  $U$  of the control space. Without loss of generality we impose the restriction  $|u_i| \leq 1$ ,  $i = 1, \dots, m$ . Such controls are deemed admissible in terms of the considered algorithms.

The benchmarking testing case is the control of a linear system that is defined by simple ordinary differential equations

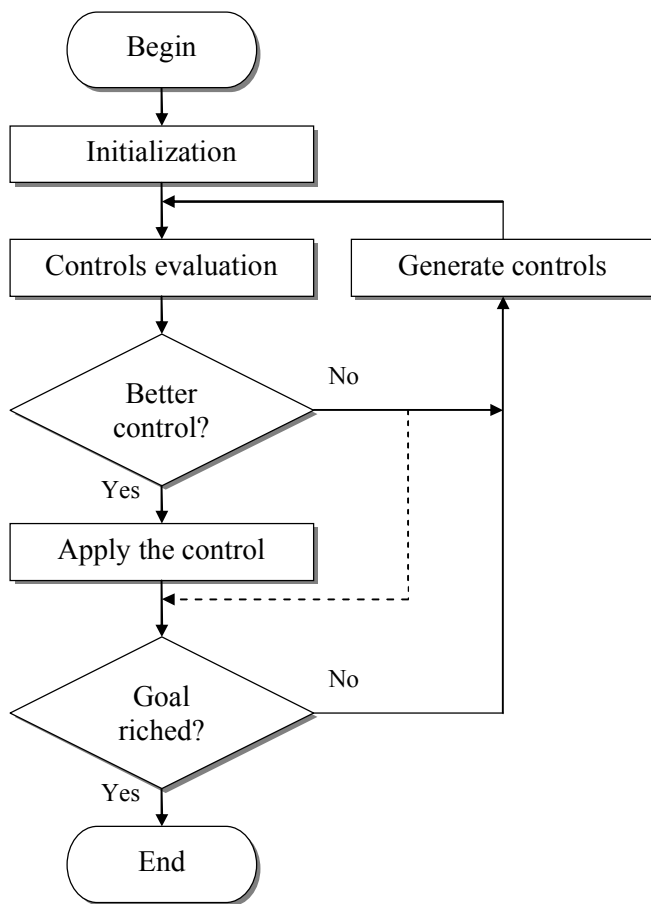
$$\frac{d}{dt} \begin{bmatrix} X \\ V \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -0.5 \end{bmatrix} \cdot \begin{bmatrix} X \\ V \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot U, \quad (1)$$

from the state  $\vec{x}_0 = \{1,1\}$  to the state  $\vec{x}_g = \{0,0\}$  with respect to the minimal transition time. Although the system (1) is simple and linear, it will be presented to the algorithm as a purely numerical model. System (1) will be controlled with presence of constraints, which are given by the following system of inequalities, defined in the state space:

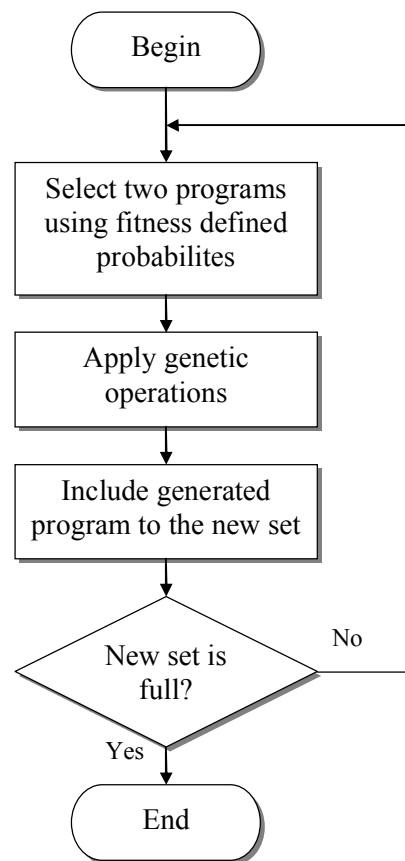
$$\begin{cases} (X - 1.5)^2 + (V - 0.5)^2 > 0.625, \\ (X - 1)^2 + (V + 0.25)^2 > 0.625. \end{cases} \quad (2)$$

The controls are allowed to have only three admissible values  $\vec{u}^0 = \{-1,0,1\}$ , and the control time step is  $\Delta t = 0.25$  s.

Algorithm structure is presented in a form of a flow-chart shown in the figure 1 and 2 below [6].



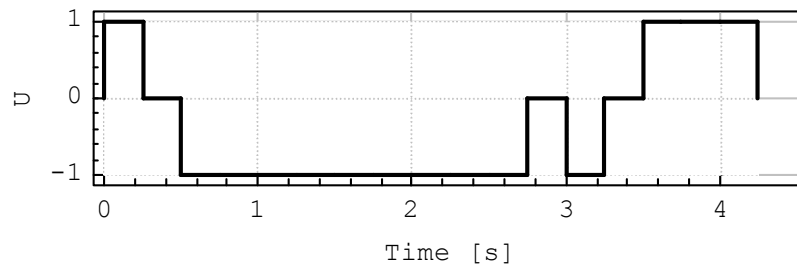
**Fig. 1.** Evolutionary control algorithm



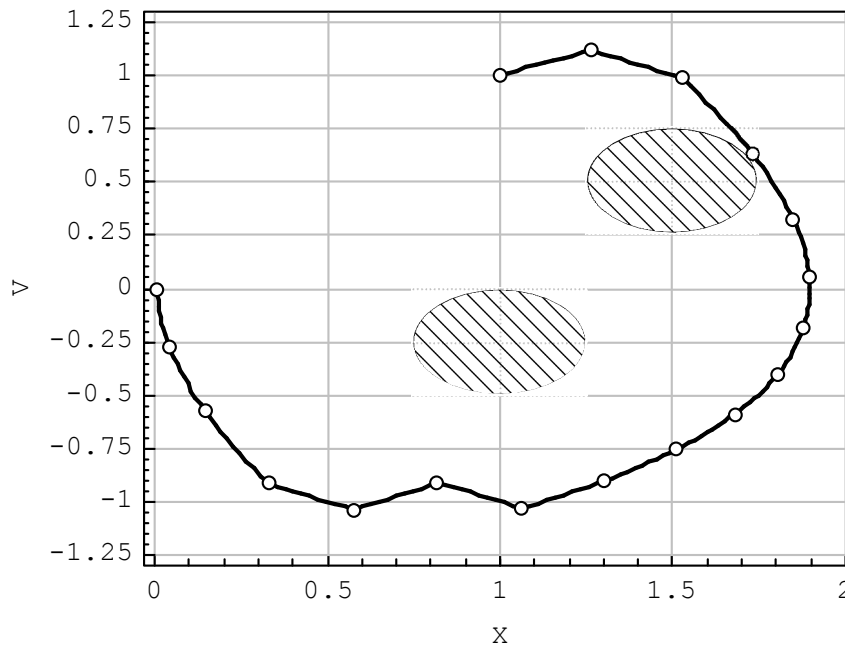
**Fig. 2.** New controls generating process

Here figure 2 depicts insides of the “generate controls” block of the flowchart in the figure 1.

Defined by the (1) and (2) control problem has known optimal solution shown in the figures 3 and 4 below. Rigorously speaking, this solution is rather sub-optimal, but the best achievable under given conditions (piecewise constant controls and fixed control time step).



**Fig. 3.** Minimal time optimal control



**Fig. 4.** Minimal time state-space trajectory with constraints

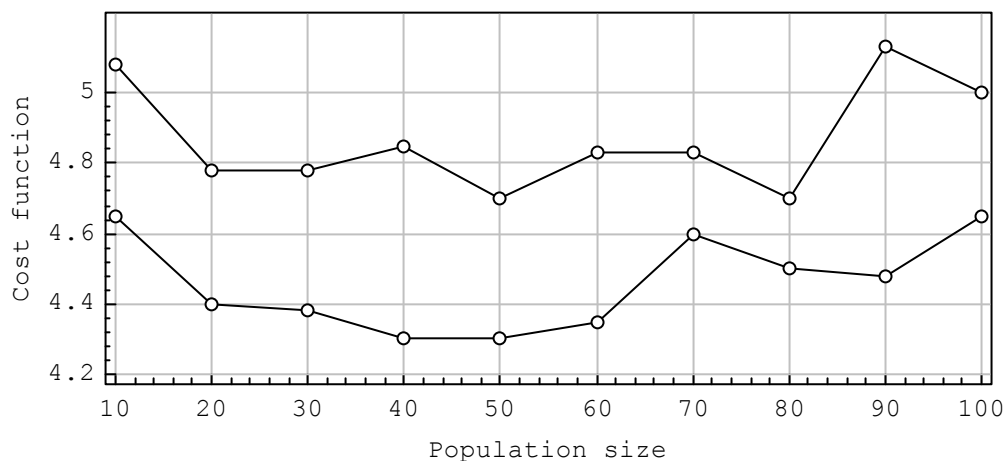
One should also distinguish acceptable solution and optimal solution. Acceptability means that control error is tolerable, but the best value for the cost function is not yet reached. Needless to say that the first one is found a lot sooner than the optimal one. In a sense, as soon as the acceptable solution is found, applying of the control function could already start.

In terms of parameters optimization, not every parameter of the algorithm is susceptible to optimisation. For example, crossover rate (probability of applying crossover) is obviously must be as high as possible (it is assumed to be 1 during the benchmarking).

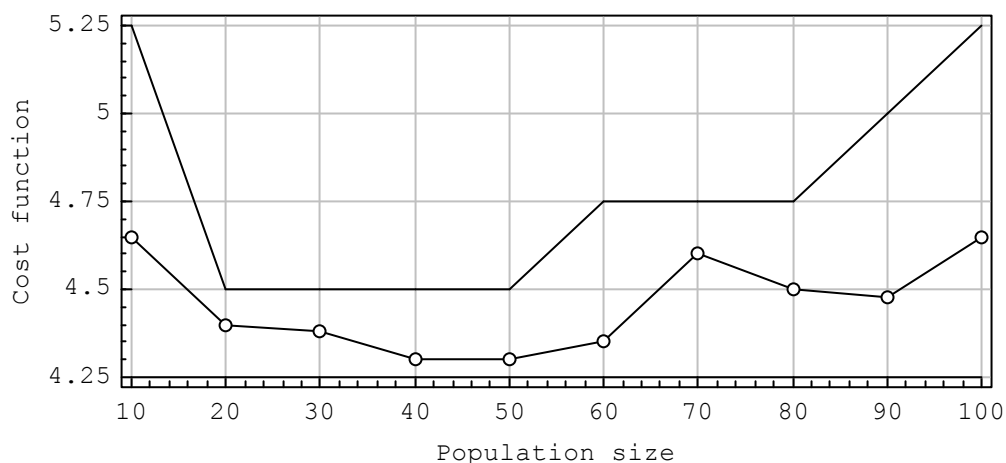
### **Population size**

Let us first analyse how performances of the algorithm are affected by the population size. Algorithm was executed 10 times for each value of the population size. As a criterion for the performances evaluation we take an averaged value of the achieved cost function for acceptable and final solutions, and averaged amount of the generations required to find these solutions. Results of the simulations are presented in the figures 5-7.

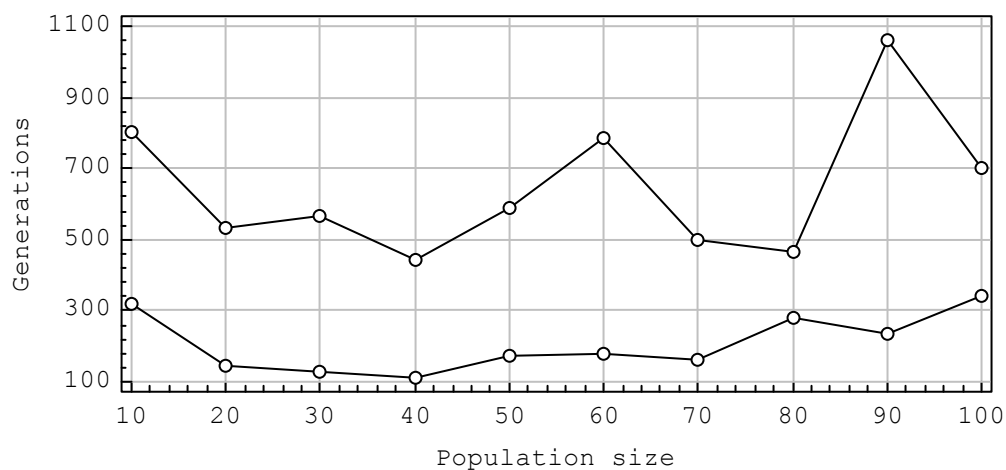
From these figures one can see that both in terms of the number of generations and achieved cost function values (the less the better) an optimal population size can be easily identified.



**Fig. 5.** Acceptable (dashed) and final (solid) values of cost function



**Fig. 6.** Averaged final value (solid), best (dotted), and worst (dashed)

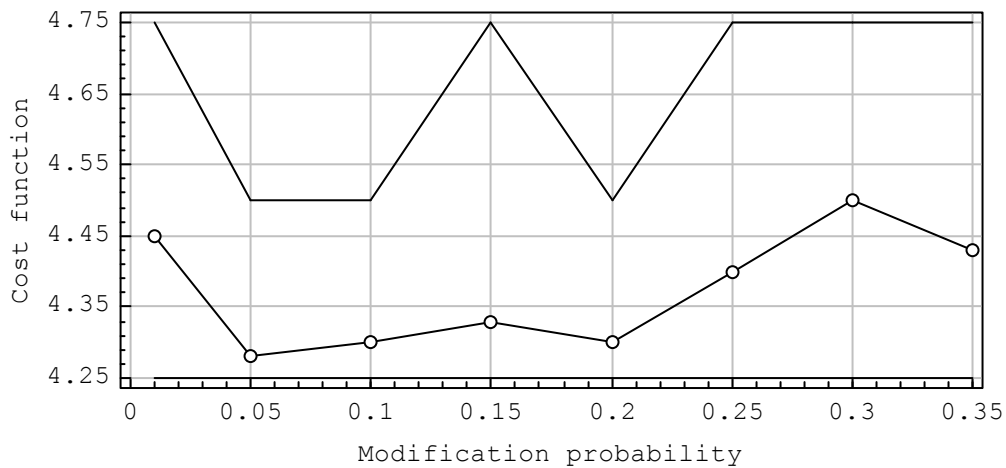


**Fig. 7.** Generations to acceptable (dashed) and best (solid) solutions

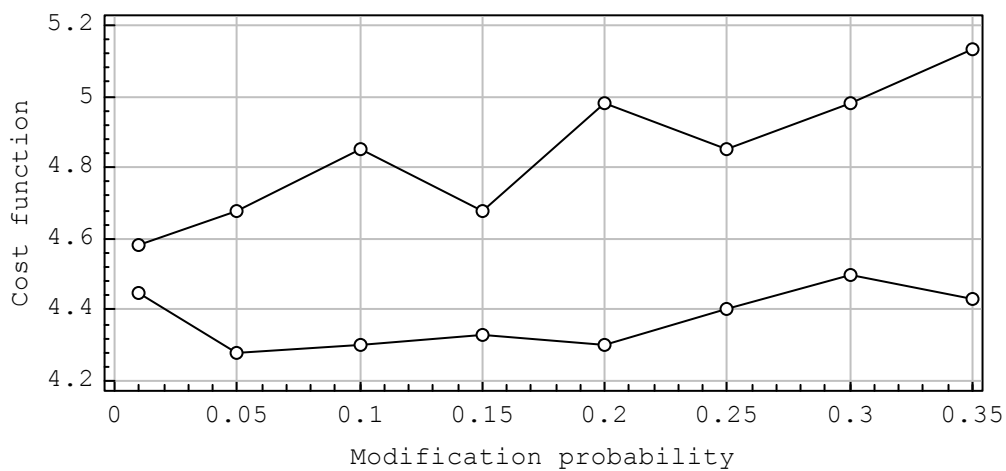
Optimal population size apparently is about **40** to **50** for the problem under consideration. While it is possible that other problems may have another optimal population size, the sheer existence of such optimality requires further investigation.

### Length modification

Another important parameter of VLGA is the rate of the length modification. This parameter explicitly affects the capability of the algorithm to find the optimal value of the unknown variable control horizon (CH). Testing results are shown in the figures 8-10 below.



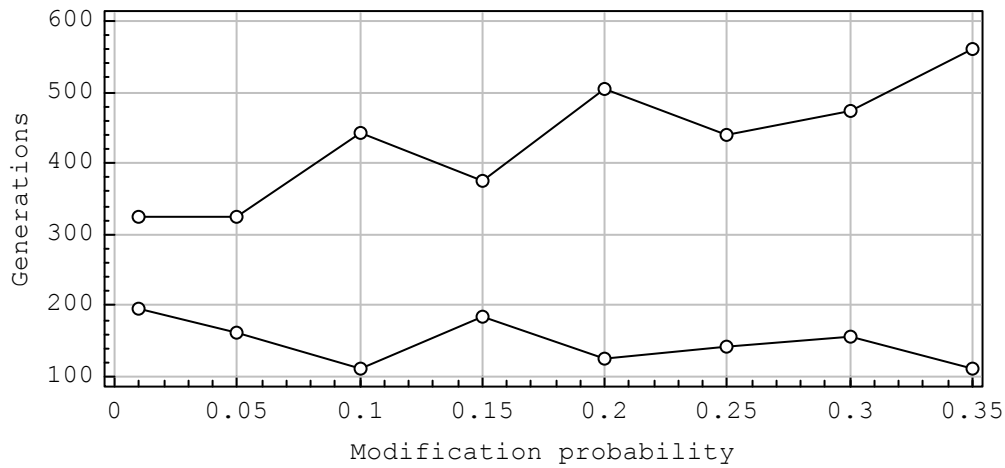
**Fig. 8.** Acceptable (dashed) and final (solid) values of cost function



**Fig. 9.** Averaged final value (solid), best (dotted), and worst (dashed)

From the analysis of these results it easy to see that high modification rates degrade performances of the algorithm while searching for the optimal solution in vicinity of the best control length. At the same time, if the current population of control functions is far from the optimal length, high values of the modification rates might be required to reach the acceptable solution.

One can see that if the averaged length is within the optimum (as is the case in these test), then modification rate **0.05-0.1** delivers the best results.



**Fig. 10.** Generations to acceptable (dashed) and best (solid) solutions

### Conclusions

Presented above study shows that proper choice of such VLGA parameters as population size and modification rate may significantly performances of the algorithm compare to previously reported results [6]. For example, in terms of the generations to best solution, 6 times improval has been achieved with the suggested above parameters.

Nevertheless, further study on dynamics of the algorithm during length search stage, and problem specific parameters optimisation are still required.

### References

- [1] Holland, J., *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor, 1975.
- [2] Goldberg, D.E., *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, 1989.
- [3] Onnen, C., Babuska, R., Kaymak, U., Sousa, J. M., Verbruggen, H. B., and Isermann, R., *Genetic Algorithms for Optimization in Predictive Control*, Control Engineering Practice, 1997, v. 5(10), pp.1363–1372.
- [4] Haber R., Schmitz U., and Bars R., *Optimal choice of horizons for predictive control by using genetic al gorithms*, Chin. J. Comput. Phys. 28, 2004, pp. 53-58.
- [5] V. Apostolyuk, *Application of State-Space Search Algorithms to Optimal Control*, Proceedings of VI Intl. Conference on Gyro Technology, Navigation, and Motion Control, Vol. 2, Kiev, 2007, pp. 104-110.
- [6] V. Apostolyuk, *Evolutionary Approach to the Procedural Optimal Control*, Electronics and Control Systems, NAU, 2008, No. 2(16), pp. 57-65.