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APPLICATION OF RECURSIVE IDENTIFICATION TO CORIOLIS VIBRATORY GYROSCOPES

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Demodulated dynamics of Coriolis vibratory gyroscopes (CVG) in difference form suitable for recursive identification is derived in this paper. CVG parameters identification framework is suggested and tested with different algorithms of recursive identification using realistic numerical simulations. Suitable algorithms for different operation conditions are identified.

Introduction. Many modern angular rate sensors operate by sensing motion induced by Coriolis force and rotation in vibrating structures. As a result, they are commonly referred to as Coriolis vibratory gyroscopes (CVG). Sensitive elements of such gyroscopes can also be fabricated in miniature form using modern micro-electronic mass-production technologies. In this case CVGs are referred to as MEMS (Micro-Electro-Mechanical-Systems) gyroscopes [1]. Either CVGs in general or MEMS in particular are often considered as a low accuracy sensors. However, necessity to expand its areas of application constantly requires improvement of different CVG performances. One of the natural ways to improve sensor performances is to use signal processing to reduce measurement noise and errors [2-5]. Unfortunately, due to the variation of CVG parameters over time, any predesigned signal processing system will eventually become irrelevant. In order to build signal processing system capable to adapt to ever changing CVG parameters, these parameters must be continuously identified during gyroscope operation. Algorithms of recursive identification are viewed as the most suitable methods to solve this problem. While problem of CVG parameters identification in modulated form is continuously addressed by researchers worldwide [6], novel approach based on the demodulated CVG dynamics is studied in this paper. Different recursive algorithms are tested and the most appropriate for different operation conditions are identified.

CVGs in systems. When used as a part of a bigger system, CVG can be represented as shown in Fig. 1.

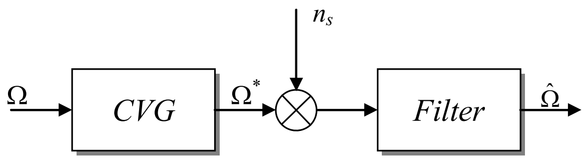

Fig. 1. CVG as a system component

Here n_s is the sensor (measurement) noise, Ω is the actual angular rate, Ω^* is the angular rate measured by the CVG, $\hat{\Omega}$ is the estimated (filtered) angular rate, which in ideal case is equal to the actual angular rate Ω.

Dynamics of the CVG sensitive element in the Fig. 1 can be sufficiently accurately described by the following approximation [4]:

$$\dot{\Omega}_* = -k\zeta\Omega_* + k\zeta\Omega\,. \tag{1}$$

Here k is the natural frequency of primary and secondary oscillations, which assumed to be equal, ζ is the relative damping of primary and secondary oscillations. Sensitive element is also assumed to be excited in primary resonance.

This simplified equation still represents observed dynamics of CVG with acceptable for most applications accuracy, thus allowing synthesis of efficient sensor and process noise filters, angular rate estimators, etc., [4].

Let us now represent system (1) in the difference state space form

$$\begin{cases} X_n = F \cdot X_{n-1} + w_{n-1}, \\ Y_n = H \cdot X_n + v_n. \end{cases} \tag{2}$$

Here X_n is the sampled state vector, Y_n is the measured state vector, H is the state measurement matrix, w_n and v_n are the process and sensor noises respectively.

State transition matrix F can be calculated from the system matrix A using inverse Laplace transformation L^{-1} [7] as

$$F = L^{-1}\{(I \cdot s - A)^{-1}\}\,.$$

State vector X and system matrix A for the equation (1) are

$$X = \{\Omega_* \quad \Omega\}',\ A = \begin{bmatrix} -k\zeta & k\zeta \\ 0 & 0 \end{bmatrix}. \tag{3}$$

Corresponding state trasition and measurement matrices in system (2) are as follows:

$$F = \begin{bmatrix} e^{-k\zeta t} & 1-e^{-k\zeta t} \\ 0 & 1 \end{bmatrix},\ H = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{4}$$

While state-space representation (2) is oftenly used in state estimation algorithms, difference equation is preferable in recursive identification algorithms, which can easily be written using components of the state transition matrix in (4):

$$\Omega_n^* = e^{-k\zeta t}\Omega_{n-1}^* + (1-e^{-k\zeta t})\Omega_{n-1} = -a \cdot \Omega_{n-1}^* + b \cdot \Omega_{n-1}\,. \tag{5}$$

In generalised form, system (5) can be represented as

$$\begin{aligned} \theta_n &= \theta_{n-1}\,, \\ \Omega_n^* &= \varphi^T \cdot \theta_n + e_n\,, \end{aligned} \tag{6}$$

where

$$\theta = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -e^{-k\zeta t} \\ 1-e^{-k\zeta t} \end{bmatrix} \tag{7}$$

is the vector of system parameters to be identified, and

$$\varphi_n = \begin{bmatrix} -\Omega_{n-1}^* \\ \Omega_{n-1} \end{bmatrix} \tag{8}$$

is the vector of delayed system input and output measurements. System (6) along with its components (7) and (8) is now suitable for applying conventional recursive identification algorithms.

CVG identification system. Analysing CVG representation (6) one should notice that input angular rate Ω present in the vector (8) is apparently unknown. However, using Kalman filter defined by (2) and (4) one will obtain estimate of the vector X containing both filtered CVG output Ω^* and estimated input angular rate $\hat{\Omega}$, which approximates unknown input angular rate Ω. These two angular rates can be used to identify CVG dynamics parameters (7) as shown in Fig. 2.

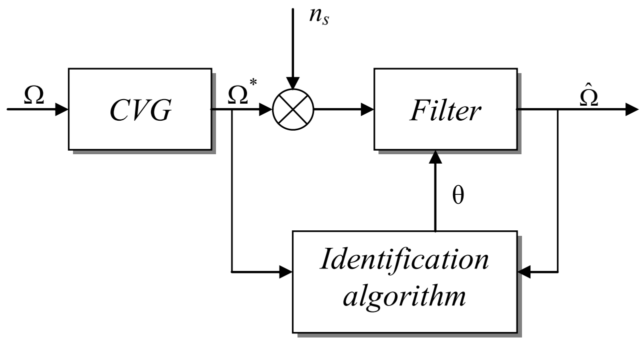

Fig. 2. CVG parameters identification

Alternatively, unfiltered noisy CVG output can also be used as a measured system output Ω^*. After system parameters vector θ is identified, it is used to update components of the trasition matrix F incorporated into the angular rate estimation filter.

Identification algorithms testing framework. Let us now test different algorithms of recursive identification in estimating CVG parameters. In order to simulate CVG operation along with filtering and identification, realistical numerical model of CVG implemented in Simulink/Matlab will be used. Angular rate has shape of squared pulses with 1 rad/s amplitude. White noise is added to the output rate prior to be fed to the Kalman filter block. All these signals are shown in Fig. 3.

Recursive identification algorithms performance will be evaluated using relative error of parameters identification expressed in percents as

$$E = \frac{a - e^{-k\zeta t}}{e^{-k\zeta t}} \cdot 100\% \, . \tag{9}$$

Here a is the system pole identified by the algorithm, t is the simulation sampling rate.

All algorithms will be tested under the following three conditions: idealistic – real angular rate as an input and clear gyro output, realistic – estimated input and filtered output, and noisy – estimated input and noisy output. The latter is used to evaluate sensitivity of the algorithm to estimation errors. The following recursive identification algorithms will be evaluated: auto-regressive with external input (ARX), auto-regressive moving average with external input (ARMAX), and Box-Jenkins algorihm (BJ) [8].

Algorithms testing results. Plotted outputs of some cases are shown in Fig. 4 and 5. Algorithms performances evaluated using (9) are presented in the Table 1.

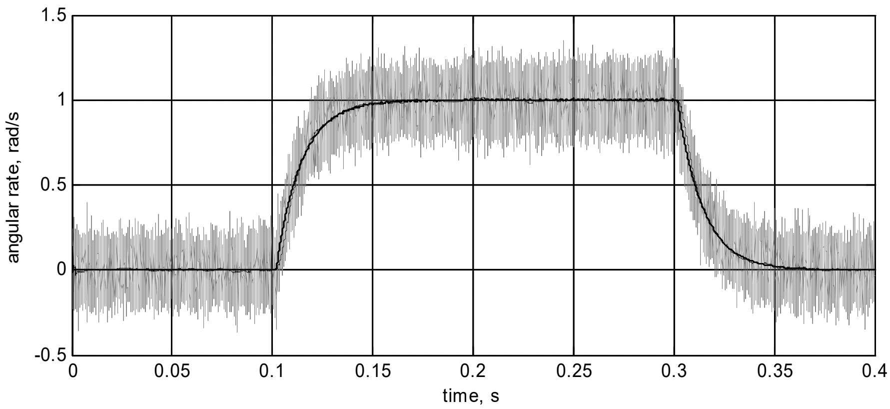

Fig. 3. Angular rate measurements
(gray – noised output, dotted – actual output without noise, solid – output estimation)

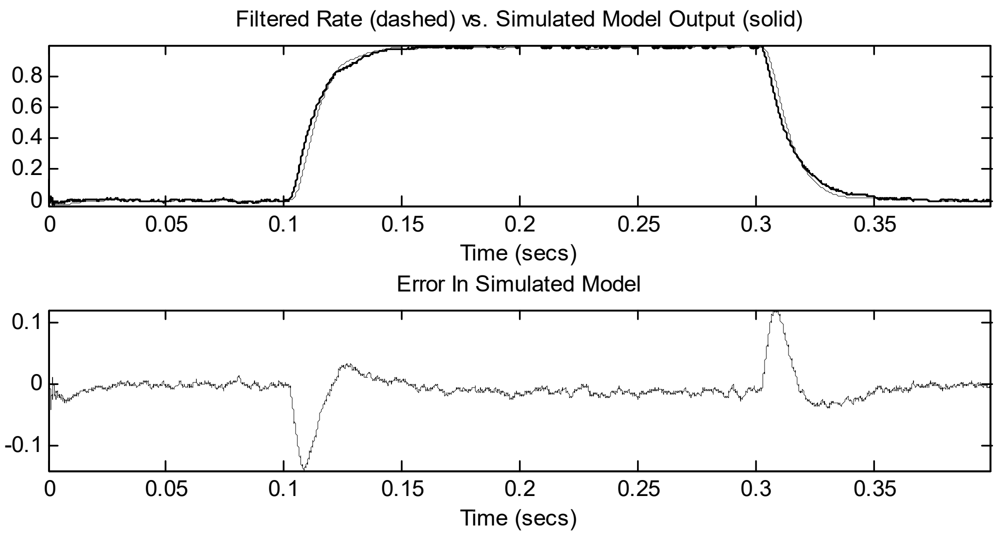

Fig. 4. ARX under realistic conditions

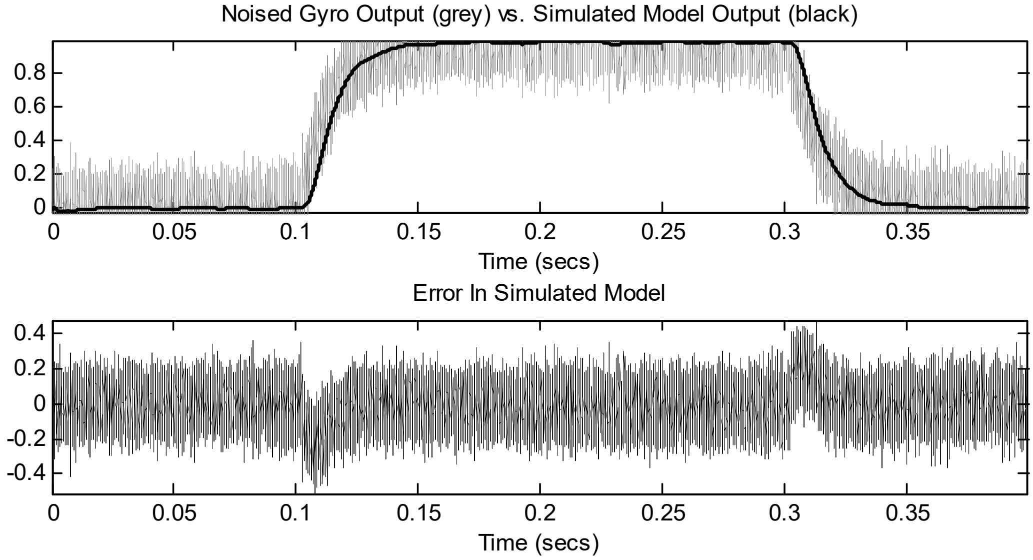

Fig. 5. ARMAX under noisy conditions

Table 1. Recursive identification algorithms performances

Algorithm	**Idealistic**	**Realistic**	**Noisy**
ARX	- 0.0175	-0.0285	-46.4500
ARMAX	0.0185	-0.0295	-0.0175
Box-Jenkins	-0.0355	0.0095	-0.0405

Conclusions. Studying presented in Table 1 performances of the recursive identification algorithms one can see that in case of high angular rate estimation errors ARMAX algorithm delivers best results, while in case of low estimation errors Box-Jenkins algorithm can be successfully applied. At the same time all algorithms demonstrated sufficiently good performances within proposed in this paper identification framework. Bearing in mind that estimated input angular rate is used in identification, and indentified parameters are later used to obtain better estimation of this input rate, overall convergence of such system should be studied in greater details. The latter viewed as topic for future research.

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